Raffles Mathematical Olympiad 2022
Round 1

Date: 29 March 2022 **Duration: 1 hour**

This paper consists of 20 questions.
*For practice purpose, the multiple choice options are removed.

The marks allocation is as follows:

Question Number	Correct	Unanswered	Incorrect
1 to 10	4 marks	1 mark	0 mark
11 to 20	6 marks	1 mark	0 mark

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1. If n is the smallest positive integer greater than 3 such that

$$\left(1+\frac{1}{3}\right)\times\left(1+\frac{1}{4}\right)\times\left(1+\frac{1}{5}\right)\times...\times\left(1+\frac{1}{n}\right)$$

is greater than 2022, find the sum of digits of n.

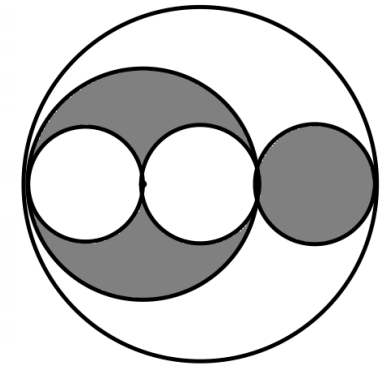
2. It is given that $\frac{1}{13}=0.076923076923076923.....$, where the first 6 digits after the decimal point 076923 repeat itself infinitely. If $\frac{23}{130}$ is written as a decimal, what is the 2022nd digit after the decimal point?

3. If the nine-digit number $\overline{123abc789}$ is divisible by 999, find the value of $a+b+c$.

4. Calculate $\frac{2022}{674+674^2}+\frac{2022}{675+675^2}+\frac{2022}{676+676^2}+...+\frac{2022}{1010+1010^2}$.

5. Calculate $\frac{8088}{24}+\frac{8088}{40}+\frac{8088}{60}+\frac{8088}{84}+\frac{8088}{112}+\frac{8088}{144}+\frac{8088}{180}+\frac{8088}{220}+\frac{8088}{264}$.

6.



In the diagram above, there are five circles with 3 different diameters and some of the circles touch each other as shown. If the diameter of the largest circle is 21 cm, taking $\pi=\frac{22}{7}$, find the area of the unshaded parts in cm^2.

7. Let $N = 2022 + \frac{2022}{1+2} + \frac{2022}{1+2+3} + \frac{2022}{1+2+3+4} + \ldots + \frac{2022}{1+2+3+4+\ldots+2021}$.

Find the sum of digits of N.

8. Find the sum of all the thirty 4-digit numbers in the pattern below.

2022	2023	2024	2025	2026	2027
2023	2024	2025	2026	2027	2028
2024	2025	2026	2027	2028	2029
2025	2026	2027	2028	2029	2030
2026	2027	2028	2029	2030	2031

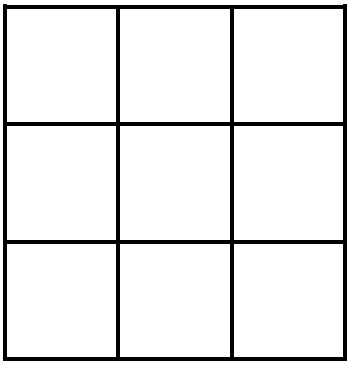
9. In RIPMWC held in 2016, the number of male contestants increased by 20% and the number of female contestants increased by 80% compared to that held in 2015. If the total number of contestants for RIPMWC held in 2016 increased by 32% compared to that held in 2015, what fraction of the total number of contestants was the number of female contestants in 2016?

10. Mary and Esther are riding bicycles around a circular track. Their starting points are the two endpoints of a diameter of the circular track. They start cycling at the same time and at constant speeds. If both of them ride anti-clockwise, they will meet after 80 minutes. If Esther rides anti-clockwise while Mary rides clockwise, they meet after 10 minutes. Given that Esther's speed is 36 km/h which is greater than Mary's speed, find perimeter of the circular track in km.

11. Some drinking water is distributed to a village. It is known that every villager drinks the same amount of water every day. If each villager drinks 1 litre less per day, the water distributed can last for 15 more days. If each villager drinks 1.5 litres less per day, the water distributed can last 30 more days. How many days can the water distributed last?

12. Eric and Jasmine both determine the colour of the clothes to wear according to the date of the day. Eric will divide the date by 8 and wear red clothes if the remainder is 0, 2, 4 or 6 and yellow clothes if otherwise. Jasmine will divide the date by 6 and wear yellow clothes if the remainder is 0 or 3, green clothes if the remainder is 1 or 4 and blue clothes if the remainder is 2 or 5. For example, since 27 leaves a remainder of 3 when divided by both 8 and 6, both Eric and Jasmine will wear yellow clothes on 27th of January. For how many days in 2022 will Eric and Jasmine wear clothes of different colours?
[Note: there are 365 days in 2022]

13.



A fixed 3×3 grid is to be filled with the integers 1, 2, 3, …, 8, 9, using each integer exactly once, in such a way that for every cell which is filled with an odd integer, all its adjacent cell are filled with even integers, and for every cell filled with an even integer, all its adjacent cells are filled with odd integers. In how many ways can this be done?
[Note : 2 cells are adjacent if they share a common edge]

14. In how many ways can one choose 3 numbers among first 30 consecutive positive integers 1, 2, 3, 4, …, 27, 28, 29, 30 so that their sum is divisible by 3?

15. Three rulers A, B and C were inserted vertically into a water tank filled with water such that one end of each ruler touches the base of the tank. Initially, the ratio of the lengths of A, B and C above the water level was 1 : 2 : 4. After adding some water, the depth of water doubled. At that time, the ratio of the lengths of A, B and C above the water level became 1 : 3 : 7. If the length of A is 20 cm, find the height of the water level in the water tank, in cm, before adding some water.

16. In a triangle ABC shown below, D is a point on AB and F is a point on AC and E is a point inside the triangle such that DE is parallel to AC and EF is parallel to AB. Given that $AF = 6$ cm, $AC = 33$ cm, $AD = 7$ cm, $AB = 26$ cm and the area of parallelogram $ADEF$ is 14 cm^2, find the area of triangle ABC in cm^2.

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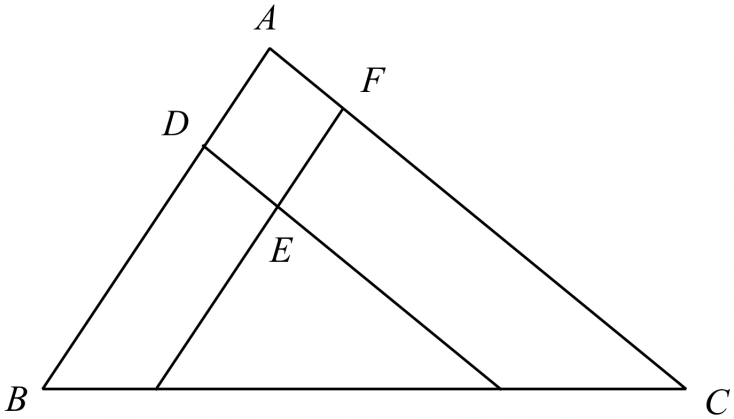
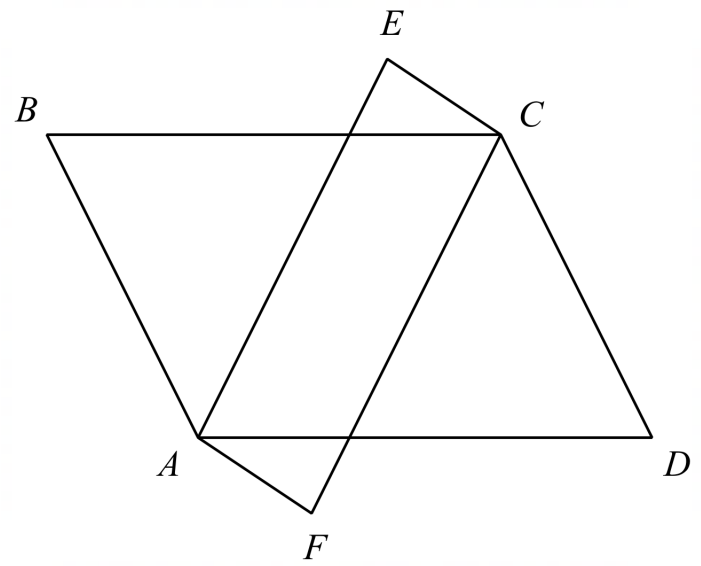


17. Find the value of

$$\frac{1}{5}+\frac{1}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\frac{5}{5^5}+\frac{8}{5^6}+\ldots\ ,$$

where the numbers in the numerators of the sum are consecutive terms of the sequence 1, 1, 2, 3, 5, 8, 13, …, in which the first 2 terms are 1 and each subsequent term is the sum of the 2 terms just before it.

18.



In the diagram above, *ABCD* and *AECF* are parallelograms and $AC = AD = CF$.
If $\angle BCF = 46^\circ$, find $\angle ECD$.

19. 1000 children are labelled 1 to 1000. Each of them has 20 candies at the beginning. Candies are given or taken away from them as follows. 3 candies are given to each child whose label is a multiple of 1, then 1 candy is taken away from each child whose label is a multiple of 2, then 3 candies are given to each child whose label is a multiple of 3, then 1 candy is taken away from a child whose label is a multiple of 4 and so on until finally 1 candy is taken away from each child with a label which is a multiple of 1000. How many children have more than 20 candies in the end?

20. A vending machine sells stamps of the values \$1.40, \$1.80, \$2.40 and \$3. If Ken needs exactly \$12 to send a parcel, how many different combinations of values are possible for the stamps he buys?

~~ The End ~~~