Raffles Mathematical Olympiad 2023
Round 1

Date: 28 March 2023 **Duration: 1 hour**

This paper consists of 20 questions.
*For practice purpose, the multiple choice options are removed.

The marks allocation is as follows:

Question Number	Correct	Unanswered	Incorrect
1 to 10	4 marks	1 mark	0 mark
11 to 20	6 marks	1 mark	0 mark

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1. If $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}$ is greater than $\frac{1823}{2023}$, find the smallest possible integer value of n.

2. A whole number has 2023 digits which are all 9. What is the sum of the digits when this number is multiplied to itself?

3. Evaluate

$$\left(\frac{1}{17}+\frac{2}{39}+\frac{3}{53}+\frac{4}{79}+\frac{5}{98}\right)\left(\frac{2}{39}+\frac{3}{53}+\frac{4}{79}+\frac{5}{98}+\frac{6}{119}\right)$$
$$-\left(\frac{1}{17}+\frac{2}{39}+\frac{3}{53}+\frac{4}{79}+\frac{5}{98}+\frac{6}{119}\right)\left(\frac{2}{39}+\frac{3}{53}+\frac{4}{79}+\frac{5}{98}\right).$$

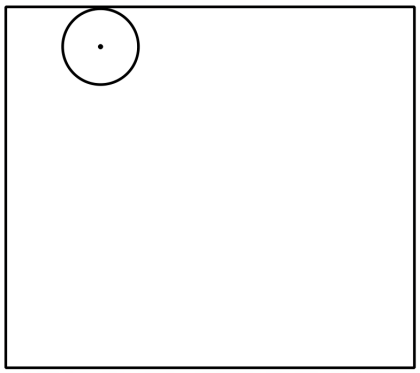
4. Given that $\underbrace{20232023...2023}_{k \text{ copies of } 2023}1965$ is divisible by 55, find the least possible value of k.

5. Find the integer part of $\dfrac{1}{\frac{1}{2013}+\frac{1}{2014}+\frac{1}{2015}+\cdots+\frac{1}{2023}}$.

6. Among the 40 colored marbles that Jack has, the ratio of blue to green marbles is 4: 1, and the number of red marbles is an integer multiple of green marbles. Jill has only 22 blue marbles and if both of their marbles are combined, the number of blue marbles is five times that of red marbles. How many marbles that belong to Jack is neither blue, red, nor green?

7. Bob and Charlie's monthly incomes is in the ratio 4 : 3 and their corresponding monthly expenses is in the ratio 11 : 7. At the end of the year, each of them saved \$9000. How much does Bob earn monthly?

8. The diagram (not drawn to scale) below shows a circle with radius 1 cm touching the interior of a square of side 10 cm. The circle is to be revolved around the sides of square and return to its original position. Find the total area that circle would cover. (Take $\pi = 3.14$).



9. Four distinct primes a, b, c and d satify the equation $ab^c c^d + a = 2023$. Find the value of $ab + bc + cd + da$.

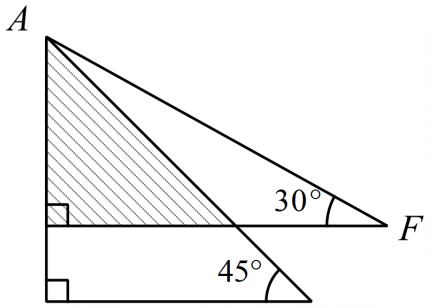
10. n students took part in a sports event which lasted k days, where $k > 1$. At the end of each day, each student is awarded a different score, namely, 1, 2, 3, …, n points, respectively. After k days, it was discovered that each student has the same accumulated score of 40 points. Given that k is a factor of n, find the value of n.

11. Alice and Bob started walking from X and Y, respectively, to Y and X. Alice's walking speed was 1.5 times that of Bob's and both of them turned back once they reached each other's starting point. If the distance between the first and second time that they met was 350 m apart, find the distance XY, in metres.

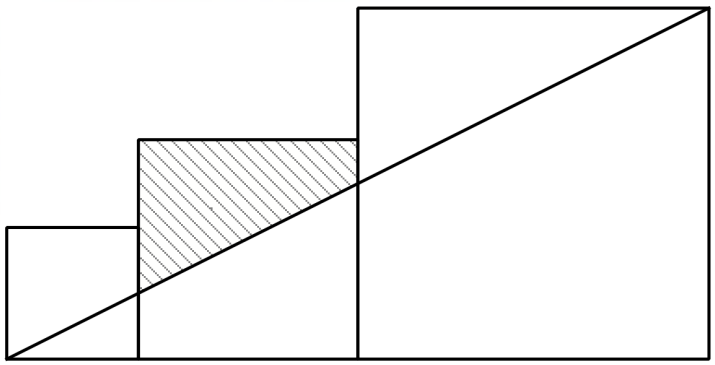
12. Village A has 600 people, where 30% are males. Village B has 400 people, where 60% are males. After some people moved from village B to A, village A now has 40% males, while village B still has 60% males. How many people moved from village B to A?

13. In a class of 46 students, 40 of the students can ride bicycles, 38 of them can play table-tennis, 35 of them can play badminton and 27 of them can swim. What is the least number of students who can play all these four sports?

14. The diagram below shows two right angled triangles, where the overlapping area is 578 cm^2. Find the length of the side AF.



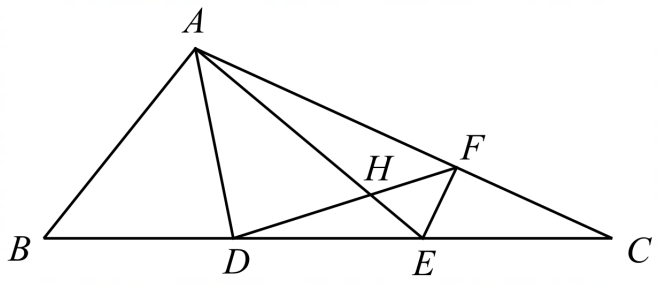
15. In the diagram below, three squares of sides 3 cm, 5 cm and 8 cm are arranged as shown. Find the area of the shaded region.



16. In an MCQ quiz, there are 10 questions each worth 3 marks, 10 questions each worth 4 marks and 5 questions each worth 6 marks, with a total maximum score of 100 marks. There is no mark deduction for unanswered or incorrect attempt. What is the number of total marks which are not obtainable in this quiz?

17. A fruit wholesaler has 23 boxes of apples and 18 boxes of oranges to distribute to 5 supermarkets. Each supermarket requires at least 4 boxes of apples and 3 boxes of oranges. How many different ways are there for the wholesaler to distribute the goods to the supermarkets? We can assume that the boxes of apples are all identical, as are the boxes of oranges.

18. In ΔABC, D and E are on BC such that $BD = DE = EC$. F is on AC such that $CF : AC = 1 : 3$. AE and DF meet at H. If the area of ΔADH exceeds that of ΔEFH by 12.6 cm^2, find the area of ΔABC.



19. In the table below, starting with the pattern 1, 2 in Row 1, the sum of which, is 3, is inserted in the new pattern in Row 2. To obtain the pattern in Row 3, the sum of 1 and 3 is inserted between 1 and 3 and the sum of 3 and 2 inserted between 3 and 2. The pattern in Row 4 is shown as follows.

Row No.	Pattern
1	1, 2
2	1, 3, 2
3	1, 4, 3, 5, 2
4	1, 5, 4, 7, 3, 8, 5, 7, 2

Find the sum of all the entries in Row 9.

20. Four distinct digits are arranged to form the largest and smallest 4-digit numbers. Given the sum of these two numbers is 11359, find the difference between these two numbers.

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