Raffles Mathematical Olympiad 2024
Round 1

Date: 26 March 2024 **Duration: 1 hour**

This paper consists of 20 questions.
*For practice purpose, the multiple choice options are removed.

The marks allocation is as follows:

Question Number	Correct	Unanswered	Incorrect
1 to 10	4 marks	1 mark	0 mark
11 to 20	6 marks	1 mark	0 mark

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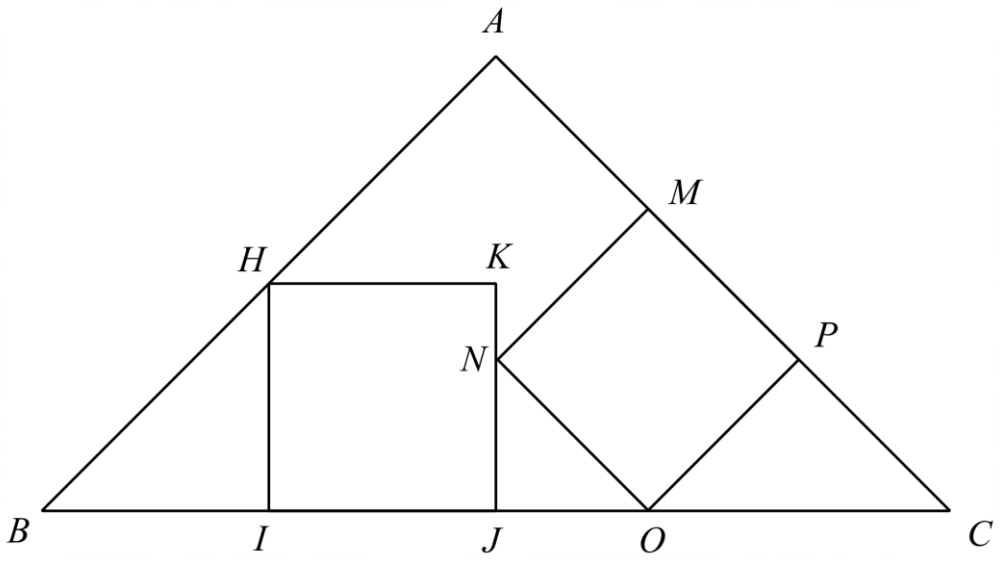
1. Multiples of 8 from 8 to 2024 inclusive are strung to form a large number $x = 8162432\ldots200820162024$. How many digits are in x?

2. In 2023, there were 30 more boys than girls in the school band. In 2024, the number of band members increase by 10%, of which the number of girls increased by 20% while the number of boys by 5%. How many members are there in the school band in 2024?

3. A bag contains 19 cards labelled 1 to 19. What is the minimum number of cards to draw at random from the bag so that we can always find two cards whose numbers add up to 20?

4. In the figure below, triangle *ABC* is a right isosceles triangle and has two squares *HIJK* and *MNOP* inscribed. Given that *AKN* is a straight line, and area of square *HIJK* is 126 cm^2, find the area, in cm^2, of square *MNOP*.



5. The sum of k consecutive positive integers is 2024. Find the greatest possible value of k.

6. Consider the following sequence

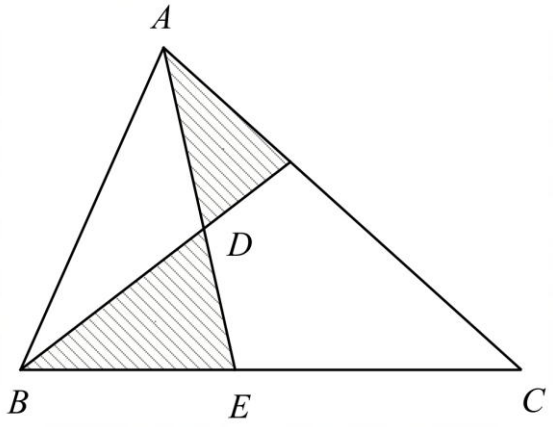
$$9,\ 24,\ 69,\ 204,\ 609,\ x,\ y,\ \ldots$$

Find the value of $x + y$.

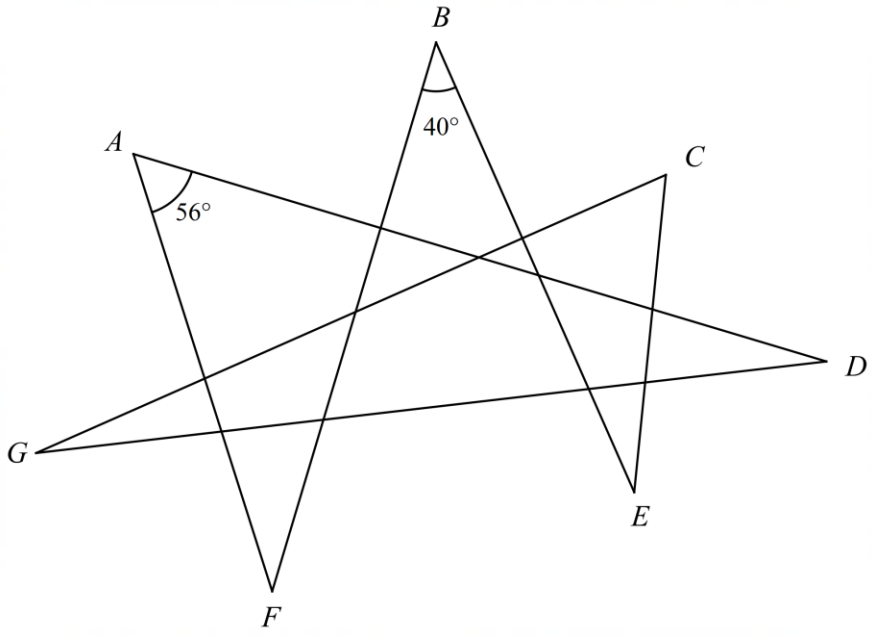
7. Alice started driving along a straight road from X towards Y while Bob started driving from Y towards X at the same time. At the start, Alice increased her speed by 15% while Bob increased his speed by 12 km/h. They met at the same spot had they kept to their original speeds. What was Bob's original speed, in km/h?

8. Three bottles of fruit juice of the same volume contain fruit concentrate and water. The ratio of the volume of fruit concentrate to that of water in the three bottles are 2:1, 4:3 and 7:5 respectively. The contents of the three bottles are transferred to a container. What is the ratio of the volume of fruit concentrate to that of water in the container?

9. In the diagram below, the area of the shaded region is 30 cm^2. Given that $AD = \frac{1}{2}AE$ and $BE = \frac{3}{7}BC$, find the area, in cm^2, of triangle ABC.



10. In the diagram, $\angle A = 56°$ and $\angle B = 40°$. Find $\angle C + \angle D + \angle E + \angle F + \angle G$.



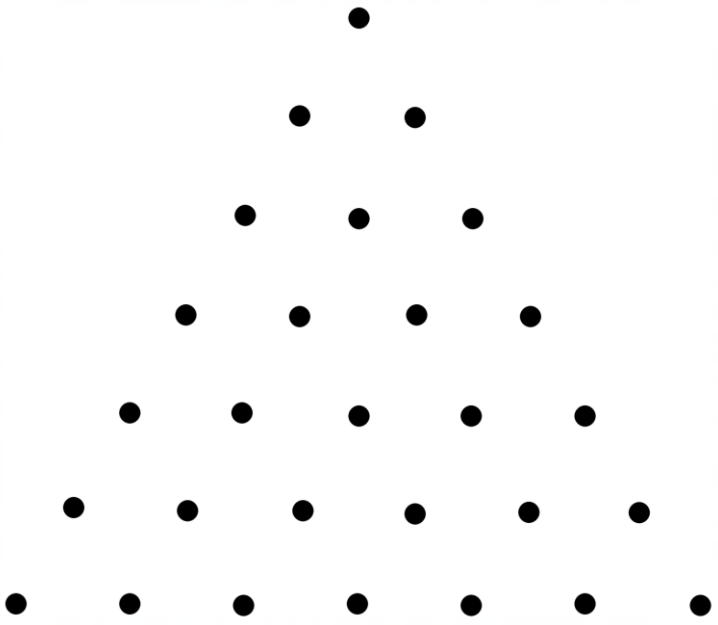
11. Let $N = \frac{1}{2^2-1} + \frac{1}{4^2-1} + \frac{1}{6^2-1} + \ldots + \frac{1}{2024^2-1}$, expressed as a fraction in the simplest form. Find the sum of all digits of the numerator and denominator of *N*.

12. Suppose a_n represents the remainder when 7^n is divided by 11. The first three terms a_1, a_2 and a_3 are given by 7, 5 and 2 respectively. Find the sum of $a_1 + a_2 + a_3 + \ldots + a_{2024}$.

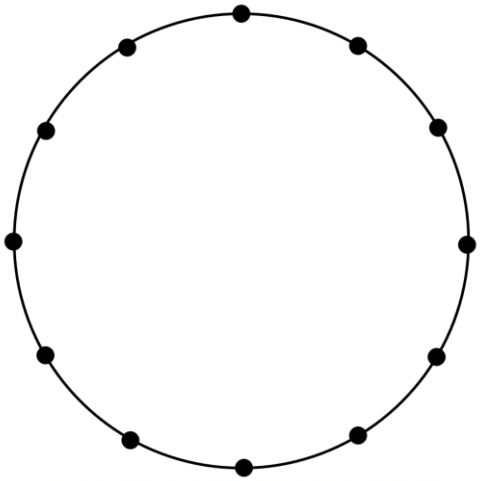
13. Find the value of
$$\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{31}\right)+\left(\frac{2}{3}+\frac{2}{4}+\frac{2}{5}+\ldots+\frac{2}{31}\right)+\left(\frac{3}{4}+\frac{3}{5}+\ldots+\frac{3}{31}\right)+\ldots+\left(\frac{29}{30}+\frac{29}{31}\right)+\frac{30}{31}.$$

14. Alice, Bob and Charlie have some pens. If Alice gives 20 pens to Charlie, then the ratio of pens of Charlie to that of Alice and Bob is 2: 1. If Alice gives 30 pens to Bob, then the ratio of pens of Bob to that of Alice and Charlie is 3: 1. How many pens do they have in total?

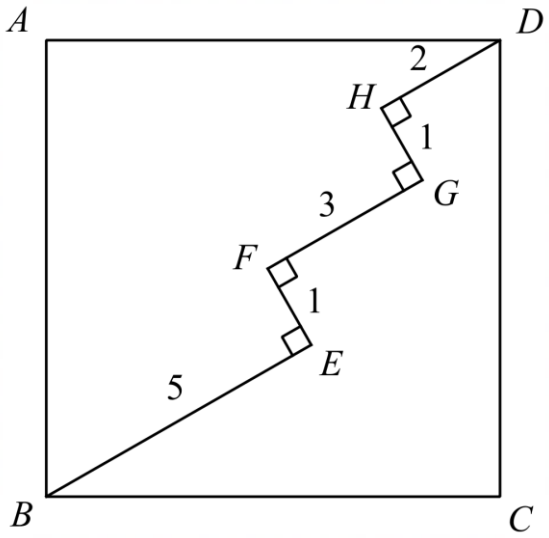
15. An equilateral triangular grid of dots is set out as shown. How many straight line segments can be drawn that pass through exactly three dots?



16. In the diagram, twelve points are equally spaced on a circle. How many different triangles can be drawn by connecting the points on the circle? Different triangles are not obtained by rotation or symmetry.

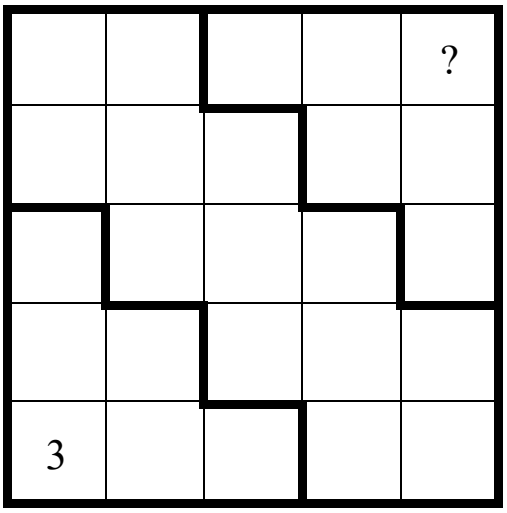


17. In the diagram, a zigzag path *BEFGHD* with four right angle turns crosses the square *ABCD*. The length of the path, in cm, is as shown on the diagram. What is the area of the square *ABCD*, in cm^2?



18. To compute $\frac{x+y}{z}$, where *x, y* and *z* are positive integers, Christopher keys in $x + y \div z$ on a calculator and obtains 15. He then keys in $y + x \div z$ and finds the answer is 20 instead. If the calculator is programmed to calculate divisions before addition, what is the correct result of $\frac{x+y}{z}$?

19. Each row and column in the following 5×5 square grid is to be filled with numbers 1, 2, 3, 4 and 5. Moreover, the sum of each of the three blocks must be equal. It is given that the bottom left entry is 3. What is the number on the top right?



20. Let $[1,2,3,\ldots,n]$ denote the lowest common multiple of 1, 2, 3, …, and n, where n is a positive integer. How many different values are there in the set

$$\frac{[1,2]}{1\times 2}, \frac{[1,2,3]}{1\times 2\times 3}, \frac{[1,2,3,4]}{1\times 2\times 3\times 4}, \ldots, \frac{[1,2,\ldots,100]}{1\times 2\times 3\times \ldots \times 100}?$$

~~~ THE END ~~~
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