Raffles Mathematical Olympiad 2025
Round 1

Date: 10 April 2025 **Duration: 1 hour**

This paper consists of 20 questions.
*For practice purpose, the multiple choice options are removed.

The marks allocation is as follows:

Question Number	Correct	Unanswered	Incorrect
1 to 10	4 marks	1 mark	0 mark
11 to 20	6 marks	1 mark	0 mark

---

1. The product $5\times 7^3\times 35$ can be expressed as x^y, where x and y are positive integers. Find the smallest value of $x+y$.

2. Calculate $\dfrac{\left(3^2+5^2+\ldots+2025^2\right)-\left(2^2+4^2+6^2+\ldots+2024^2\right)}{\left(3+5+\ldots+2025\right)-\left(2+4+6+\ldots+2024\right)}$.

3. Write one "1", followed by two "2"s, three "3"s, four "4"s, …., ten "10"s, eleven "11"s, twelve "12"s, and so on. What is the digit in the 2025$^{\text{th}}$ position?

4. The total marks of twenty-five students in a test is 2025 and no two students have the same mark. Among them, the highest mark obtained is 94 and all marks are integer values. Find the smallest possible mark.

5. How many 3-digit positive integers are there such that the tens digit is the average of the other digits?

6. Find the number of trailing zeroes in the product

$$2025\times 2024\times 2023\times\ldots\times 3\times 2\times 1.$$

7. Calculate

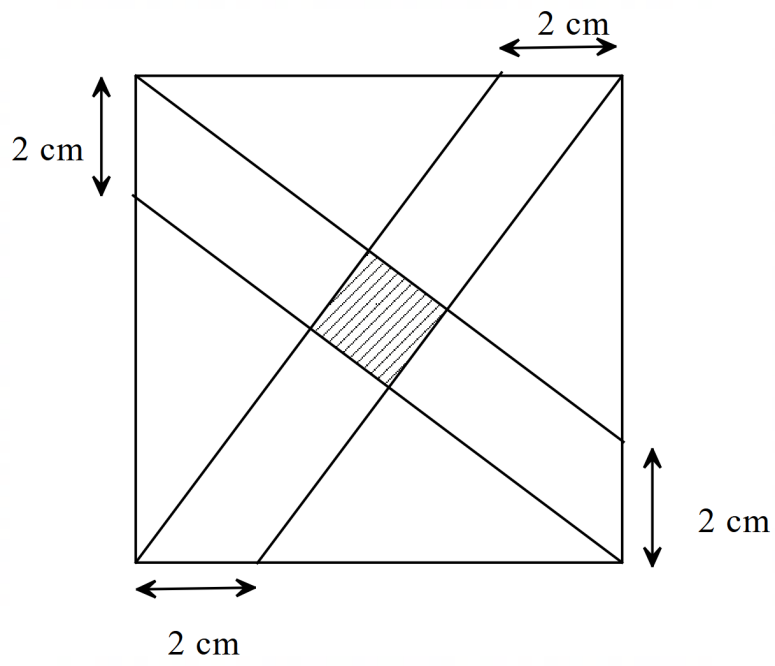
$$\left(1+\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2023}{2025}\right)\left(\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2021}{2023}\right)$$
$$-\left(\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2023}{2025}\right)\left(1+\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2021}{2023}\right).$$

8. Four students Alice, Bob, Charlie and Diana know the number of marbles that each of them have. Each of them have different number of some marbles.

Alice:	I have neither the most nor the least.
Bob:	I do not have the least.
Charlie:	I have the most.
Diana:	I have the least.

Exactly one of the four students is lying. Who is lying?

9. In the diagram, the length of one side of the outer square is 8 cm. A smaller shaded square is constructed as shown. What fraction of the area of the outer square is shaded?



10. Calculate $\frac{1008}{24}+\frac{1008}{40}+\frac{1008}{60}+\frac{1008}{84}+\frac{1008}{112}+\frac{1008}{144}+\frac{1008}{180}+\frac{1008}{220}+\frac{1008}{264}$.

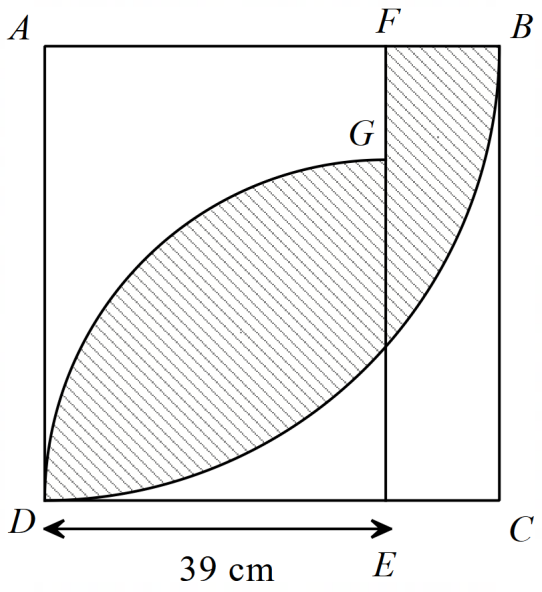
11. Calculate $\frac{1025\times 2025}{410+205^2}+\frac{1025\times 2025}{414+207^2}+\frac{1025\times 2025}{418+209^2}+\ldots+\frac{1025\times 2025}{446+223^2}$.

12. An eight digit phone number $\overline{85a32b63}$ is divisible by 99. Find the value of $11a+9b$.

13. Alice and Bob started their journey from Town X to Town Y, at 8:00 am. Alice completed the journey in 2 hours and 24 minutes while Bob took 3 hours to complete the journey. Suppose Alice made a return to Town X at the same speed immediately after she reached Town Y, at what time would Alice and Bob meet?

14. There were initially a total of 100 blue, green, red and yellow stickers in a box. The ratio of blue to green stickers was 5: 1, while the number of green stickers divides evenly that of the red stickers. After 134 blue stickers were added, the ratio of blue to red stickers became 9: 1. How many yellow stickers were there in the box?

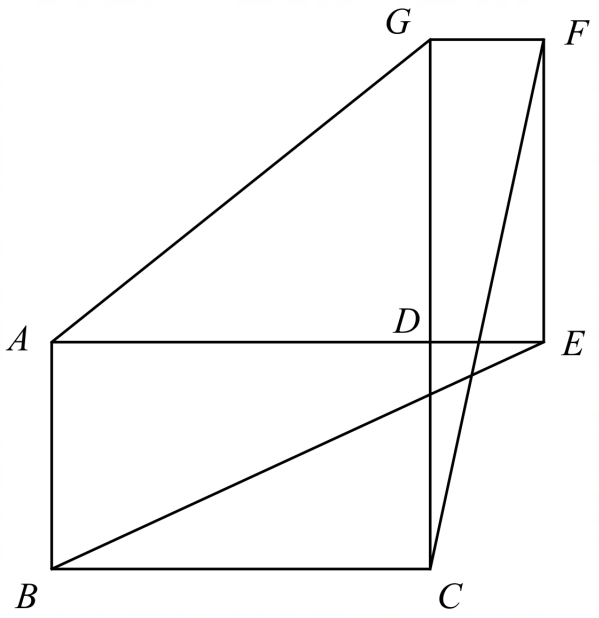
15. In the diagram, a square *ABCD* has an area of 2025 cm^2. A rectangle *ADEF* with breadth *DE* = 39 cm is drawn such that the arcs *DG*, *BD* and lines *BF*, *FG* formed a shaded region as shown. Find the area, in cm^2, of the shaded region. (Take $\pi = 3$.)



16. There are a total of 60 red and black balls. If these balls are packed in lot of tens into six boxes, then there is at least one box with more than 3 red balls. If these balls are packed in lot of twenty into three boxes, then there is at least one black ball in each box. How many black balls are there?

17. Jiahe has 42 cookies initially. He gives a prime number, *x*, of cookies to Juntao and has a prime number of cookies left. Next, he gives a prime number, *y*, of cookies to Andrew and once again, has an prime number of cookies left. Finally, he gives a prime number, *z*, of cookies to Martin and has a non-zero even number of cookies left. How many possible sets of (x, y, z) are there? Note that *x*, *y* and *z* are not necessarily distinct.

18. In the diagram, rectangles $ABCD$ and $DEFG$ are placed such that ADE and CDG form straight lines. Angle $DAG = 35°$, angle $FCG = 6°$, and $BE = CF$. Find the size of angle AEB.



19. The number of days required by Charlie and Diana individually to complete Projects X and Y are shown in the following table.

	Number of days required by Charlie	**Number of days required by Diana**
Project X	10	7
Project Y	16	20

If Charlie and Diana may work individually and/or team up for both projects, find the least number of days required for both projects to be completed.

20. A 9-digit number contains each digit from 1 through 9 exactly once. This number fulfils all the following conditions:

(I) No two consecutive digits are both even or both odd.
(II) The sum of the first 6 digits is less than the sum of the last three digits.
(III) Three perfect square digits are placed consecutively.
(IV) The 4-digit number created by taking the first four digits is divisible by 16.

What is the remainder when this number is divided by 7?

~~~ THE END ~~~
~~~