Raffles Mathematical Olympiad 2025
Round 1

Date: 10 April 2025 **Duration: 1 hour**

This paper consists of 20 questions.
*For practice purpose, the multiple choice options are removed.

The marks allocation is as follows:

Question Number	Correct	Unanswered	Incorrect
1 to 10	4 marks	1 mark	0 mark
11 to 20	6 marks	1 mark	0 mark

1.

Since $x^y = 5^2 \times 7^4$ must be divisible by 5 and 5^2 but not any higher powers, the largest y is 2. Hence $5^2 \times 7^4 = \left(5 \times 7^2\right)^2 = 245^2$ and the smallest $x + y = \boxed{247}$.

2.

$$\frac{\left(3^2 + 5^3 + \ldots + 2025^2\right) - \left(2^2 + 4^2 + 6^2 + \ldots + 2024^2\right)}{(3 + 5 + \ldots + 2025) - (2 + 4 + 6 + \ldots + 2024)}$$

$$= \frac{\left(2025^2 - 2^2\right) + \left(2023^2 - 4^2\right) + \ldots + \left(3^2 - 2024^2\right)}{(3 + 5 + \ldots + 2025) - (2 + 4 + 6 + \ldots + 2024)}$$

$$= \frac{(2025 + 2)(2025 - 2) + (2023 + 4)(2023 - 4) + \ldots + (3 + 2024)(3 - 2024)}{(3 + 5 + \ldots + 2025) - (2 + 4 + 6 + \ldots + 2024)}$$

$$= \frac{2027\left[(2025 - 2) + (2023 - 4) + \ldots + (3 - 2024)\right]}{(3 + 5 + \ldots + 2025) - (2 + 4 + 6 + \ldots + 2024)}$$

$$= \boxed{2027}$$

3.

For the first segment of the string of digits consisting of 1, 2, 3, ..., 9, there are $1 + 2 + 3 + \cdots + 9 = 45$ digits.

In the string that follows, with segments 10, 11, 12, ..., 45, there are $2 \times (10 + 11 + 12 + \ldots + 45) = 1980$ digits.

Taken together, there are $45 + 1980 = 2025$ digits, which means that the 2025th digit in the string has to be $\boxed{5}$.

4.

In order to find the lowest possible marks, every other student has to score as high as possible. This means that the first 24 students must score $94, 93, 92, \ldots, 71 (= 94 - 24 + 1)$ marks respectively.

Thus, the lowest possible marks is $2025 - \dfrac{24(71 + 94)}{2} = \boxed{45 \text{ marks}}$.

5.

By listing, we have

111	123	135	147	159
210	222	234	246	258
321	333	345	357	369
420				
	$\ddots$			
		$\ddots$		
951	963	975	987	999

Notice that once the hundreds digit has been chosen, we can form five integers satisfying the condition. Thus, there are a total of $9\times 5=\boxed{45}$ positive integers.

6.

Since $10=2\times 5$ and the number of multiples of 2 exceeds that of 5, then we may count the number of factor 5 in

$$2025\times 2024\times 2023\times \ldots \times 3\times 2\times 1.$$

Using $[x]$ to denote the integer part of x, we see that

$2025\times 2024\times 2023\times \ldots \times 3\times 2\times 1$ has $\left[\frac{2025}{5}\right]+\left[\frac{2025}{5^2}\right]+\left[\frac{2025}{5^3}\right]+\left[\frac{2025}{5^4}\right]$

factors of 5, i.e.

$2025\times 2024\times 2023\times \ldots \times 3\times 2\times 1$ has $405+81+16+3=\boxed{505}$ trailing zeroes.

7.

Let $x=\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2021}{2023}$.

Then

$$\left(1+\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2023}{2025}\right)\left(\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2021}{2023}\right)$$
$$-\left(\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2023}{2025}\right)\left(1+\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\ldots+\frac{2021}{2023}\right)$$
$$=\left(1+x+\frac{2023}{2025}\right)(x)-\left(x+\frac{2023}{2025}\right)(1+x)$$
$$=x(1+x)+\frac{2023}{2025}x-x(1+x)-\frac{2023}{2025}(1+x)$$
$$=\boxed{-\frac{2023}{2025}}$$

8.

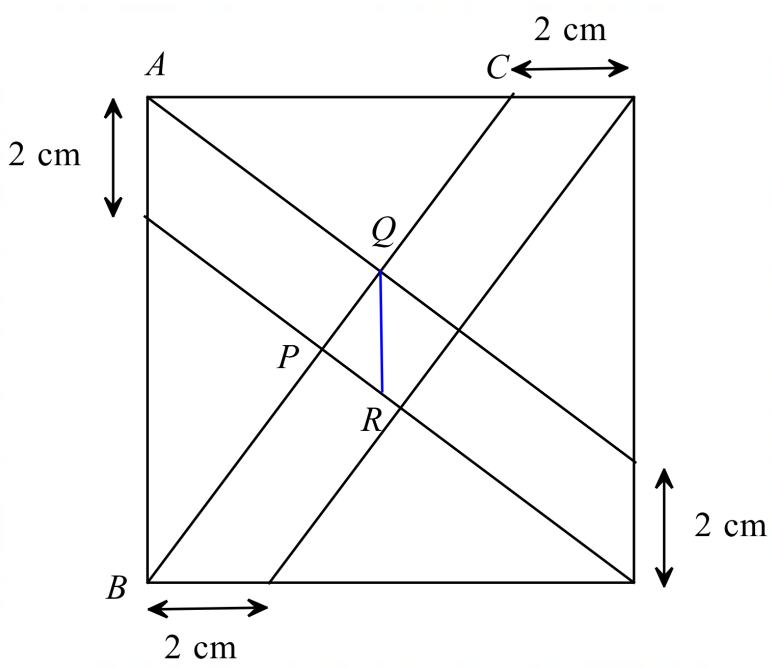
Suppose Alice is lying, then either Charlie or Diana must also be lying, meaning more than one student is lying. So Alice must be telling the truth.

Suppose Bob is lying, then Diana is also lying, so Bob must be telling the truth.

Suppose Diana is lying, then none of Diana, Alice or Bob have the least, so Charlie must have the least. This would mean Charlie would be lying, so Diana must be telling the truth.

Thus, Charlie must be lying and he does not have the most. This is consistent with Bob having the most, then Alice and Charlie in some order and Diana has the least.

9.



We construct a vertical line segment within the small square and label the vertices as shown.

By Pythagoras Theorem, we have $BC = 10$ cm.

Then since $\angle ABC = \angle PQR$ (alt. $\angle$s) and $\angle BAC = 90° = \angle QPR$,

we have triangles ABC and PQR are similar.

So, area of shaded square $= PQ^2$

$$= \left(QR \times \frac{AB}{BC}\right)^2$$

$$= \left(2 \times \frac{8}{10}\right)^2$$

$$= \frac{64}{25} \text{ cm}^2$$

and required fraction is $\boxed{\frac{1}{25}}$.

10.

$$\frac{1008}{24}+\frac{1008}{40}+\frac{1008}{60}+\frac{1008}{84}+\frac{1008}{112}+\frac{1008}{144}+\frac{1008}{180}+\frac{1008}{220}+\frac{1008}{264}$$

$$=252\times\left[\frac{1}{6}+\left(\frac{1}{10}+\frac{1}{15}\right)+\left(\frac{1}{21}+\frac{1}{28}\right)+\left(\frac{1}{36}+\frac{1}{45}\right)+\left(\frac{1}{55}+\frac{1}{66}\right)\right]$$

$$=252\times\left[\frac{1}{6}+\frac{1}{6}+\frac{1}{12}+\left(\frac{1}{20}+\frac{1}{30}\right)\right]$$

$$=252\times\left(\frac{1}{3}+\frac{1}{12}+\frac{1}{12}\right)$$

$$=252\times\frac{1}{2}$$

$$=\boxed{126}$$

11.

$$\frac{1025\times2025}{410+205^2}+\frac{1025\times2025}{414+207^2}+\frac{1025\times2025}{418+209^2}+\ldots+\frac{1025\times2025}{446+223^2}$$

$$=1025\times2025\times\left(\frac{1}{205\times207}+\frac{1}{207\times209}+\frac{1}{209\times211}+\ldots+\frac{1}{223\times225}\right)$$

$$=1025\times2025\times\frac{1}{2}\times\left(\frac{1}{205}-\frac{1}{207}+\frac{1}{207}-\frac{1}{209}+\ldots+\frac{1}{223}-\frac{1}{225}\right)$$

$$=25\times41\times81\times25\times\frac{1}{2}\times\left(\frac{1}{205}-\frac{1}{225}\right)$$

$$=\frac{1}{2}\times25\times(5\times81-41\times9)$$

$$=\frac{1}{2}\times25\times(36)$$

$$=\boxed{450}$$

12.

Let's say that 99 divides $\overline{85a32b63}$ can be written as $99 \mid \overline{85a32b63}$.

Thus, $9 \mid \overline{85a32b63}$ means $9 \mid (8+5+a+3+2+b+6+3)$

i.e. $9 \mid a+b$

so $a+b=9$ or 0 (rej. as $11 \nmid 85032063$)

and $11 \mid \overline{85a32b63}$ means $11 \mid (3+b+3+5)-(6+2+a+8)$

i.e. $11 \mid b-a-5$

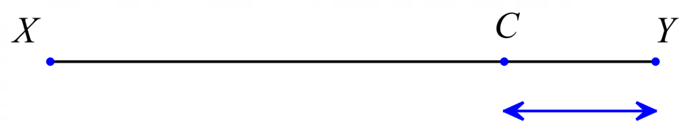
so $b-a=5$ or -6 (rej. as no integer soln possible*)

Solving $a+b=9$ and $b-a=5$ yields $a=2$ and $b=7$,

whence $11a+9b=\boxed{85}$.

*$(a,b)=(0,6),\ (1,7),\ (2,8)$ or $(3,9)$ would not satisfy $a+b=9$.

13.



Let the distance between XY be x km and suppose Bob reached C at the instance when Alice arrived at Town Y, i.e. at 10:24 am.

So, Alice's speed $= \dfrac{x}{2\frac{24}{60}} = \dfrac{5}{12}x$ km/h

while Bob's speed $= \dfrac{1}{3}x$ km/h

Moreover, distance covered by Bob $= XC$

$= \dfrac{1}{3}x \times 2\dfrac{24}{60}$

$= \dfrac{4}{5}x$ km

which means the remaining distance to Town $Y = CY$

$= x - \dfrac{4}{5}x$

$= \dfrac{1}{5}x$ km

and their combined speed $= \dfrac{5}{12}x + \dfrac{1}{3}x = \dfrac{3}{4}x$ km/h

From this point onwards, time required before they meet

$= \dfrac{x}{5} \div \dfrac{3x}{4} = \dfrac{4}{15}$ h $= 16$ min

Thus, they would meet at $\boxed{\text{10:40 am}}$.

14.

Suppose there are n green stickers, where n is integer. Then the ratio of blue to green to red stickers is $5n:n:kn$, where k is integer.

Now, $\dfrac{134+5n}{kn}=\dfrac{9}{1}$

$$(9k-5)n=134$$

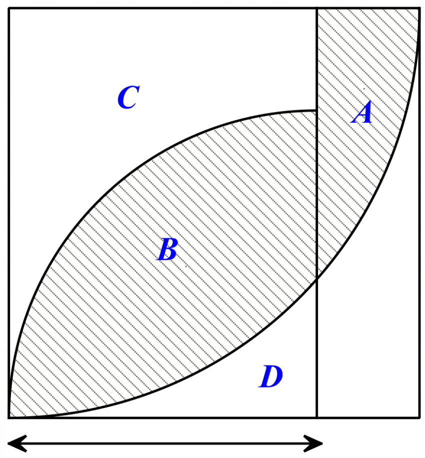
Since the factors of 134 are $1, 2, 67$ and 134,

if $9k-5=1$, 2 or 134, then k is not an integer,

if $9k-5=67$, then $k=8$, and $67n=134$ so $n=2$.

Thus there are 2 green, 10 blue and 16 red stickers initially, which means there are $100-2-10-16=\boxed{72\text{ yellow stickers}}$.

15.



Since area of square $=2025=3^4\times5^2$, side of square $=45$ cm

Label the regions as shown in the diagram.

Area of shaded region

$=A+B$

$=(A+B+C)-C$

$=(A+B+C)-[(B+C+D)-(B+D)]$

$=(A+B+C)+(B+D)-(B+C+D)$

$=\dfrac{1}{4}\times3\times45^2+\dfrac{1}{4}\times3\times39^2-45\times39$

$=\dfrac{3}{4}\times(45^2+39^2)-45\times39$

$=\dfrac{3}{4}\times(84^2-2\times45\times39)-45\times39$

$=5292-4387.5$

$=\boxed{904.5\text{ cm}^2}$

16.

Since packing in lot of ten into six boxes, there is at least one box with more than 3 red balls, by pigeonhole principle, there are at least $3\times 6+1=19$ red balls. However, when packing in lot of twenty in three boxes, there is at least one black ball in each box, we know that there are at most 19 red balls. This means that there are exactly 19 red balls, and so there are exactly $\boxed{\text{41 black balls}}$.

17.

Since Jiahe has an odd prime number of cookies left after giving two prime numbers of cookies, either x or y must be 2. But x cannot be 2, since $42-2=40$ is not prime, so y must be 2. Testing small primes, we find that only $x=11,\ 23$, or 29 satisfy the conditions for x and y.

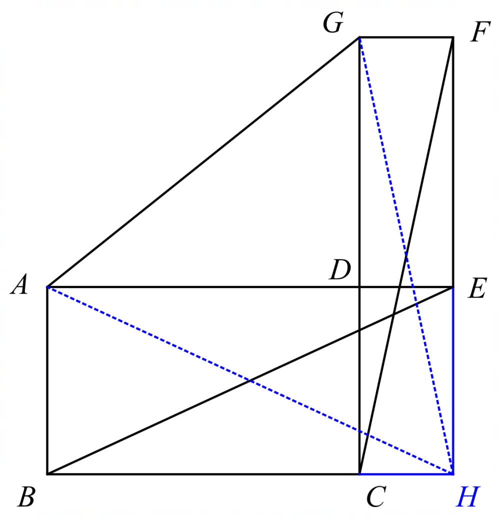
For $x=11$ and $y=2$, z can be 3, 5, 7, 11, 13, 17, 19 or 23.

For $x=23$ and $y=2$, z can be 3, 5, 7, 11, or 13.

For $x=29$ and $y=2$, z can be 3, 5 or 7.

In total, we have $\boxed{16}$ sets of (x,y,z).

18.



Produce BC and FE to meet at H.

Then $ABHE$ and $CHFG$ are rectangles for which $AH=BE$ and $CF=GH$.

Since $BE=CF$, we must have $AH=GH$.

Thus, ΔAHG is isosceles, and $\angle GAH=\angle AGH$.

Furthermore, $\angle CGH=\angle FCG=6°$ by property of diagonals of rectangle.

Now, $\angle AEB=\angle HAE$

$$=\angle AGH-\angle DAG$$

$$=(180°-90°-35°+6°)-35°$$

$$=\boxed{26°}$$

19.

We note that Diana is more efficient in completing Project X alone, while Charlie is more efficient in completing Project Y alone, so for a start, let Diana work on Projects X individually and Charlie on Y.

After 7 days, Project X is completed while

fraction of Project Y remaining $= 1 - \frac{7}{16} = \frac{9}{16}$

In one day, fraction of Project Y that both Charlie and Diana can complete

$$= \frac{1}{16} + \frac{1}{20} = \frac{9}{80}$$

Thus, number of remaining days required $= \frac{9}{16} \div \frac{9}{80} = 5$ days

Therefore, Charlie and Diana need minimally $\boxed{12 \text{ days}}$ to complete both projects.

20.

From (II), we immediately derive that the last 3 digits are either 6,8,9 or 7,8,9.

By (I), however, only 7,8,9 is possible.

By (III), we must have either 149 or 941 appear in the number, and hence we derive that the last five digits are 14987.

By (IV), we only need to check how to arrange 2,3,5,6 to form a multiple of 16.

In this case, only 5632 works. Thus, the number is 563214987.

Whence, the remainder of 563214987 upon division by 7 is $\boxed{6}$.